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Traffic control and route choice: modelling and optimisation

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1. Introduction

The main purpose of this paper is to encourage the consideration and development of new automatic control systems which deliver better utilisation of limited network capacity (taking account of drivers' and travellers' route choices). Such control systems may be expected to actively encourage "economical" routeing patterns; these are routeing patterns which consume less of the scarce commodity in congested urban networks today. The scarce commodity in congested networks is "junction capacity".

The central idea here is to make the maximum positive use of the natural dynamical system illustrated in figure 1.

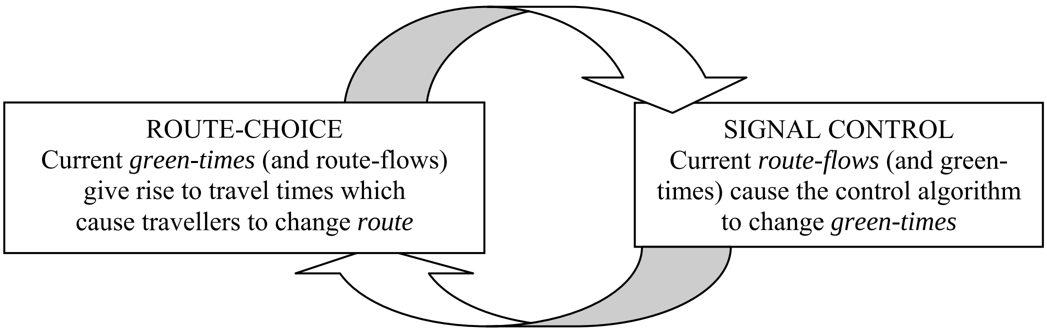

Figure 1. The dynamical system arising when a responsive control system is utilised. Current route-flows change green-times (according to some responsive control policy or algorithm) and current green-times generate delays which change route-flows (as travellers seek quicker / cheaper routes). The loop is traversed indefinitely. (Here, in this paper, we assume the signal control part of this dynamical system, the right hand box, is very quick.)

To make better use of the dynamical system illustrated in figure 1, automatic control algorithms are needed which (i) are responsive to current conditions but which also (ii) encourage beneficial, congestion-reducing route choice changes in the future. (Standard responsive control systems *are* responsive to current conditions but, in general, *do not seek to encourage congestion-reducing route-choice changes in the future*; indeed standard traffic control systems, in general, have the effect of discouraging any route-choice changes at all, by seeking to almost always "do the best" for traffic flows as they currently are; rather than as they might be tomorrow.)

Thus this paper addresses the following question: *is there a responsive control algorithm which may be utilised within the dynamical system illustrated in figure 1 to systematically encourage congestion-reducing routeing changes in the future?*

There is a second purpose to this paper; this second purpose has two elements:
(i) to contribute to the development of new traffic assignment and control models which take some reasonable account of the spatial development of queues, including blocking back; and
(ii) to tie such blocking-back modelling advances to the main aim of this paper, namely to advance the design of responsive control systems which systematically encourage more efficient travel choices.

This second purpose is a major and exciting long term project that will probably prove to be very challenging. In this paper we illustrate one such modelling development; this results in a model suitable for modelling traffic control and route choice *even when there are short lanes*.

Much of this paper is indicative since the scope of the paper is very wide; the technicalities described merit much more consideration by engineers and mathematical modellers. However some detail is given; especially of the link model which allows for blocking back. The need for such models has been recently emphasised by Bliemer et al (2012) and others.

1.1. An organisational comment

To tackle the two tasks above there is a need for excellent working connections between mathematical modellers, transport and traffic software model developers and traffic control system designers.

1.2. Practical significance

These days, because there is so much information available, or becoming available, there is a powerful and understandable tendency to seek automatic traffic systems that cause the network to react effectively (and quickly) to up-to-date information concerning events.

As part of this endeavour, it is clearly important and natural to seek quickly or slowly responsive traffic control systems which are not only "good" when judged from a performance viewpoint (for a variety of scenarios) but also "good" when judged from a stability viewpoint (again for a variety of scenarios).

This paper considers both the performance and the stability of one standard automatic control response in traffic networks, and suggests one new responsive control system which may combine high performance with high stability, taking the reactions of travellers changing their routes (in response to changed conditions) into account.

1.3. Outline of the paper

Section 2 of this paper shows how very simple dynamical mathematical models and very simple thought experiments may be used to justify a special responsive signal control strategy which does take advantage of the dynamical system illustrated in figure 1 and does encourage congestion reducing routeing changes.

Then in section 3 we consider how spatial queueing may be introduced into assignment models via a simple 2-dimensional link performance model. This model allows for blocking back (and short lanes). In section 4 we show how this 2-dimensional link performance model may be amended to include signal green times and section 5 states the Wardrop equilibrium condition which allows for blocking back, by using the link performance model created in section 3. Section 7 provides a deliberately wide (but brief) context.

2. A simple example routeing / control example using the equisaturation control policy or algorithm and using the P_0 control policy or algorithm

2.1. The simple network

We consider here a simple asymmetrical network in Fig. 2 with one node signalised. The difference in uncongested route travel times is to be K minutes and is significant. Route 2 takes K minutes more to traverse than route 1, when both are uncongested.

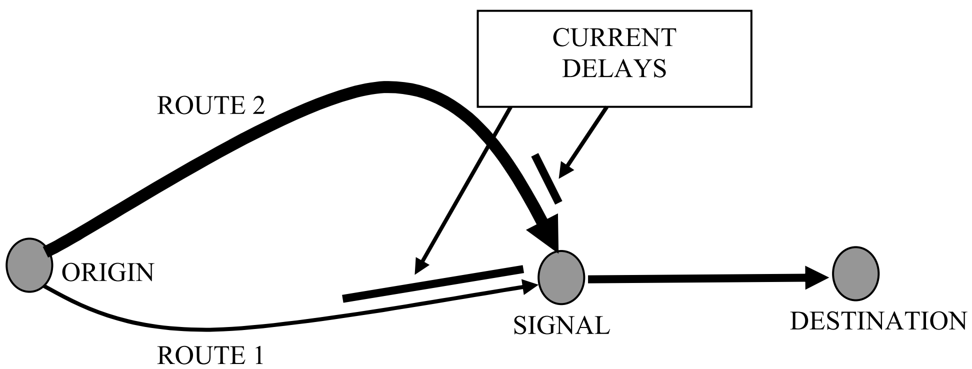

Figure 2. A simple asymmetrical signal-controlled network. Route 2 is substantially longer and (ignoring delays at the signal) route 2 travel time is substantially greater than route 1 travel time. Route 2 (or approach 2) is also twice as wide as route 1 (or approach 1) at the signal; the saturation flow s_1 of route 1 and the saturation flow s_1 of route 2 satisfy $s_2 = 2s_1$. Current average delays at the signal are represented by the lengths of bars on routes 1 and 2.

2.2. The simple network and the equisaturation policy

Suppose that the standard equi-saturation policy is utilised at the signal. (This policy or related policies are often now used as an essential part of current control systems.) Suppose also that *travellers switch over time to quicker routes if these are available.* (1)

Suppose initially that most of the origin-destination flow is using route 2. Then under natural and general conditions the travel time on route 2 will exceed the travel time along route 1. Thus, in the dynamical system illustrated in figure 1 and following dynamical system (1), flow will naturally swap from route 2 to the quicker route 1 as time passes. Since there is now more flow on route 1 or approach 1, the equisaturation policy will respond by allocating more green time to route 1 or approach 1 (and less to route 2). This causes the travel time on route 2 to still exceed the travel time on route 1 (often by a greater margin). Thus flow, again following dynamical system (1), will naturally swap from route 2 to the quicker route 1. Since there is now yet more flow on route 1 or approach 1, the equisaturation policy will again respond by allocating yet more green time to route 1 or approach 1 (and less to route 2). This causes the travel time on route 2 to still exceed the travel time on route 1 (often by an even greater margin). Travellers following (1) again swap from route 2 to route 1. And so on.

Thus, in this equi-saturation case, in general, the dynamical system illustrated in figure 1 causes flow and green-time to always swap from route 2 to route 1.

Now consider two special cases:

1. Total origin-destination flow low (= $2s_1/3$ say).
2. Total origin-destination flow high (= $3s_1/2$ say).

Recall that $s_2 = 2s_1$.

2.2.1. Case 1: total origin-destination flow low (= $2s_1/3$ say).

Suppose initially that much the greater proportion of the OD flow is initially on route 2. In this case, delays at the junction are small since congestion at the junction is small for any reasonable signal control policy (including equisaturation). Thus travel times are dominated by the uncongested travel times and the travel time on route 2 will exceed the travel time along route 1. Thus, in the dynamical system illustrated in figure 1, flow following (1) will naturally swap from route 2 to the quicker route 1 as time passes. Because delays at the signal are small route 1 will still be quicker than route 2. So more traffic swaps from route 2 to route 1. (Again, this will happen under any reasonable signal control policy including the equisaturation policy.) Under reasonable assumptions and in this low flow case, eventually all traffic will use route 1 and the green-time on route 2 will be the minimum allowed. In this low flow case the travel times experienced will progressively decline; so everyone will feel benefits of the dynamical system as time passes.

2.2.2. Case 2: total origin-destination flow high (= $3s_1/2$ say).

Again, suppose initially that much the greater proportion of the OD flow is initially on route 2. Suppose also that initially delays at the junction are small compared to the travel time difference K. In this case travel times are still initially dominated by the uncongested travel times and so the travel time on route 2 will exceed the travel time along route 1. Thus, in the dynamical system illustrated in figure 1, flow will naturally, at least initially, swap from route 2 to the quicker route 1. This will happen under any reasonable signal control policy including equisaturation. As traffic flow on route 1 increases there must in this high flow case come a time when junction delays become large compared to K.

However we are here using the equisaturation policy. So, under most reasonable assumptions on the delay formulae utilised, it may be shown that, as before, the travel time along route 2 exceeds the travel time along route 1. In this circumstance the dynamical system (1) still causes flow to switch from route 2 to the still quicker route 1.

In this high flow case the total travel time eventually increases as trajectories of the dynamical system evolve and the delays at the signal increase. In this case the saturation flow of route 1 is insufficient to cater for the total OD flow ($3s_1/2 > s_1$) and under natural conditions the loop in Fig. 1 causes the (flow, green-time) pair to converge (as time passes) to a (flow, green-time) pair *where delays become very large indeed; this standard equi-saturation policy fails to maximise the capacity of the network.*

This is very undesirable. It means that in this network the dynamical system converges to the set of states where long and increasing queues would be inevitable and quite unnecessary. While this is happening congestion at the junction progressively increases and travel times also continually increase. (The behaviour here occurs with other policies and not just the equisaturation policy. For example it also occurs with policies which seek to approximate delay-minimisation.)

Does this happen with realistic networks? How can this undesirable state of affairs be avoided? What about changing the responsive control algorithm?

2.3. The simple network and the P_0 control policy or algorithm

The counterproductive dynamics just described suggest that policies other than equi-saturation or delay-minimisation should be considered. Smith (1979a, b, 1987) introduced the following signal control policy.

In this paper s_i minutes is the saturation flow at the link i exit and b_i minutes is to be the delay at the link i exit. Consider a junction with just two approaches: link 1 and link 2. Then the P_0 policy selects green times which equalise the following two values:

$$s_1b_1 \text{ and } s_2b_2.$$

Here delays are assumed to depend on green times as well as flows (and possibly queues). Dynamically, if faced with two approaches where

$$s_1b_1 < s_2b_2$$

the policy swaps green-time from approach 1 to approach 2 until

$$s_1b_1 = s_2b_2$$

(or until a minimum green time constraint is reached and there is no more green-time available to swap). This policy utilises only local data. The effect on the previous network is now outlined in two case:

1. Total origin-destination flow low (= $2s_1/3$ say).
2. Total origin-destination flow high (= $3s_1/2$ say).

2.3.1. Low flow (case 1): total origin-destination flow = $2s_1/3$ say.

Suppose initially that much the greater proportion of the OD flow is initially on route 2. In this case, delays at the junction are small since congestion at the junction is small for any reasonable signal control policy. Thus in this case travel times are dominated by the uncongested travel times and so the travel time on route 2 will exceed the travel time along route 1. Thus, in the dynamical system illustrated in figure 1, flow will naturally swap from route 2 to the quicker route 1 as time passes. This will happen under any reasonable signal control policy including P_0. Under reasonable assumptions, eventually all traffic will use route 1 and only a minimum green time is awarded to route 2. This is just like the equisaturation case.

2.3.2. High flow (case 2): total origin-destination flow = $3s_1/2$ say.

Again, suppose initially that much the greater proportion of the OD flow is initially on route 2. Suppose also that initially delays at the junction are small compared to the travel time difference K. Thus in this case travel times are still initially dominated by the uncongested travel times and so the travel time on route 2 will initially exceed the travel time along route 1. Thus, in the dynamical system illustrated in figure 1, flow following dynamical system (1) will naturally, at least initially, swap from route 2 to the quicker route 1. This will happen under any reasonable signal control policy including P_0.

Now however, with P_0, as traffic flow on route 1 increases there must in this high flow case come a time when junction delays become large compared to K, since total OD flow exceeds s_1.

Since we are now using control policy P_0,

$$s_1b_1 = s_2b_2 \text{ or } d_1 = 2b_2.$$

Thus as soon as $b_1 > 2K$,

$$b_1 – b_2 = b_1 – b_1/2 = b_1/2 > K$$

and the travel time along route 1 exceeds the travel time along route 2. In this circumstance the dynamical system (1) causes flow to switch from route 1 to route 2, as this now has less travel time.

It is clear that the special control policy P_0 has a stabilising effect on this network and encourages economical routeing swaps when route 1 flow is high. As time passes the system converges to a quite satisfactory (flow, green-time) pair and travel times remain reasonable, unlike the equi-saturation case described in section 2.2.2. above.

2.4. Performance results for all origin-destination loads.

We are really much more interested in network performance when demand flows are not certain and not fixed at just two values. So consider the figure 2 network when the total OD flow increases slowly from 0. In this case the equisaturation policy will keep the route 2 green-time at a minimum and by far the greater flow will always be on link 1 which has a saturation flow of s_1. Thus the “equisaturation” performance will be as illustrated in figure 3. Figure 3 also illustrates in a similar fashion the performance of responsive P_0. This policy does encourage route swaps toward route 2 once the junction delays become significant compared to K. In the simplest models the equisaturation curve will have a vertical asymptote at a flow somewhat greater than s_1 and the P_0 curve will have a vertical asymptote at a flow slightly less than $s_2 = 2s_1$. (If the minimum green-times were zero then the asymptotes would be exactly at s_1 and $s_2 = 2s_1$.)

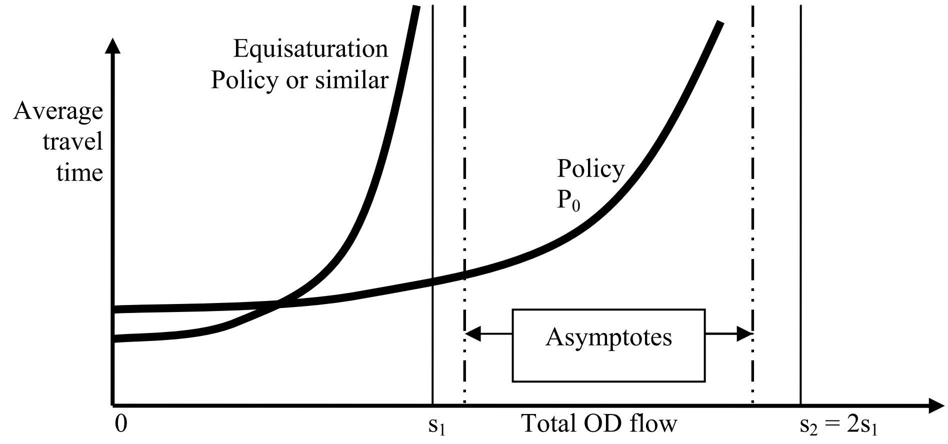

Figure 3. Equilibrium performances of responsive equisaturation and P_0 control policies as the total flow from the origin to the destination in the network in figure 2 increases from zero. The capacity of the network is nearly doubled by switching from equi-saturation or delay-minimisation to P_0.

3. A possibly new simple link performance model which allows for spatial queueing and blocking back within a traffic assignment model.

In this section a simple link performance model, developed in joint work with Huang (Katholieke Universiteit Leuven, Belgium) and Viti (University of Luxembourg, Luxembourg), is outlined. This link model allows for the spatial development of queues and so may be suitable for modelling route choice even when there are short lanes which give rise to blocking back; which often happens in modelling traffic signal control.

The link model follows Thompson and Payne (1975), extends Smith (1987, 2012) and is motivated by many papers including especially Daganzo (1998).

3.1. A link model with spatial queueing (but without blocking back)

As usual in traffic modelling, each real-life traffic lane is here represented by

1. a node which represents the entry point of the lane,
2. a node which represents the exit point or the stop line of the lane, and
3. a directed link joining these two nodes which represents the stretch of lane between the entry and the exit of the lane.

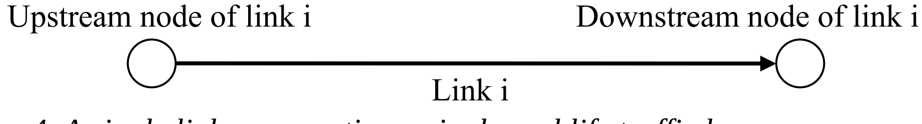

Figure 4. A single link representing a single real life traffic lane.

A representation of link i is shown in figure 4. Representations of two lanes are connected in the model by short links when traffic may, in reality, pass from one to the other and are otherwise unconnected. These additional short links represent all possible movements at junctions, from one lane to another, and junctions are thus represented in an "expanded" form.

The flow along link i is v_i vehicles per minute; the saturation flow at the exit of link i is s_i vehicles per minute; the queue at the exit of link i is Q_i vehicles; the maximum possible value of Q_i is $MAXQ_i$; and the time to traverse the entire length of link i (when the queue $Q_i = 0$ and the flow is v_i) is $c_i(v_i)$. The link i "state" may be thought of as (v_i, Q_i) and this link state 2-vector is to be confined to a set of supply-feasible pairs (v_i, Q_i), as follows:

$$v_i \leq s_i \text{ and } Q_i \leq MAXQ_i.$$

The cost / travel-time function $c_i(.)$ is to be a positive, continuous, non-decreasing function of just v_i and is defined for all non-negative $v_i \leq s_i$.

Consider a link i with a feasible flow v_i and a feasible queue Q_i and no congestion on any downstream link; so that the saturation flow s_i at the link i exit is the only constraint on the link flow. To calculate the queueing delay b_i (minutes per vehicle) at the link i exit it has often been proposed (see Thompson and Payne (1975) and Smith (1987) for example) that:

$$b_i = Q_i / s_i.$$

Here we are considering a steady state model; however this formula is also very common in the literature on dynamic traffic assignment (appearing then as: $b_i(t) = Q_i(t) / s_i$ for all times t, sometimes for just short links i). Then the whole or total time of travel on link i has often been written:

$$tt_i = c_i(v_i) + b_i = c_i(v_i) + Q_i/s_i. \tag{2}$$

This is the point queue model; where queueing is imagined to occur "vertically" at the end of the link. Suppose now that $Q_i = 0$. In this special case, where there is no queueing and so the bottleneck delay $b_i = 0$, formula (2) is entirely reasonable. Consider now the case where

$$0 < Q_i < MAXQ_i.$$

In this case the queue on link i covers part of the length of link i and only the remainder has to be traversed (with no queue). The non-zero queue will take up space on link i and so formula (2) in general overestimates the travel time for a link, double counting delays felt on that part of the link which contains the queue. This overestimate is larger when the queue is larger. The time for traversing link i will thus be the sum tt_i of a queueing component Q_i/s_i and a non-queueing component strictly less than $c_i(v_i)$. So here we put:

$$tt_i(v_i, Q_i) = (1- Q_i/MAXQ_i)c_i(v_i) + Q_i/s_i. \tag{3}$$

Formula (3) is a natural way of overcoming the double counting in (2). (Note, for example, that if $Q_i = MAXQ_i$ then the first term in (3) vanishes and the total travel time for this link is all queueing time.)

However, if we use this link model (3) then it follows that, realistically, if $v_i < s_i$, and the link outflow is regarded just here as s_i and so is unimpeded, then the queue must shrink (as the input to the queue is less than the saturation flow). Thus in a steady state assignment model at equilibrium (where queues do not change with time) we must, if we use link performance function (3), have

$$\text{either } v_i = s_i \text{ or } Q_i = 0.$$

It is clear that this whole link model (3) does not allow for blocking back; where at equilibrium a link i flow $v_i < s_i$ and also there is a non-zero (stationary) queue Q_i (which cannot dissipate since there is a queue blocking the link i exit).

To allow for this eventuality where blocking back occurs it is natural to change link performance model (3) to:

$$tt_i(v_i, Q_i) = (1- Q_i/MAXQ_i)c_i(v_i) + Q_i/v_i, \tag{4}$$

remembering also the important constraints:

$$v_i \leq s_i \text{ and } Q_i \leq MAXQ_i. \tag{5}$$

In this case, with (4) and (5), it becomes possible for the link i flow to be less than the saturation flow and the queue to be simultaneously positive. Travel times estimated by (4) and (5) are also, after a little consideration, seen to be reasonable from an engineering viewpont.

The set SB of supply-feasible base (link flow vector, queue vector) pairs is now defined as follows:

$$SB = \{(\mathbf{v}, \mathbf{Q}); \mathbf{v} \leq \mathbf{s}, \mathbf{Q} \leq \mathbf{MAXQ}\}. \tag{6}$$

4. Adding in green times.

Building on the previous section, it is now easy to add green times. We simply keep the link performance function (4) but change the constraint (5). Let g_i be the proportion of time that link i is given green. Then change constraint (5) to:

$$v_i \leq s_i g_i \text{ and } Q_i \leq MAXQ_i. \tag{7}$$

Here we are thinking of the link i green time proportion g_i as fixed. Also the set SB in (6) now becomes for fixed green-times:

$$SB(\mathbf{g}) = \{(\mathbf{v}, \mathbf{Q}); v_i \leq \mathbf{s_i} g_i \text{ for all i and } \mathbf{Q} \leq \mathbf{MAXQ}\}. \tag{8}$$

5. Equilibration without green times

The simplest equilibrium model making use of the above two-dimensional link model (4) uses routes and route flows. In such a model route travel times are obtained by adding up relevant link travel times (given by the link performance function (4)). We assume here that for each OD pair total route flows between that OD pair are fixed. Then Wardrop equilibrium is simply expressed as usual (see Wardrop (1952)):

for any pair of routes joining a single OD pair and having different travel times, the route with the greater travel time is not used.

Let **M** be the route-link incidence matrix so that

$M_{ir} = 1$ if link i is on route r and $M_{ir} = 0$ if link i is not on route r.

Let D be the set of demand feasible route flow vectors **X**. This set is the set of non-negative route flow vectors which meet the given fixed OD demands. Consider a vector of route flows **X** and a vector of queue sizes **Q**. Then the vector of link flows must be $\mathbf{v} = \mathbf{MX}$ and the travel time TT_r along route r is then given by:

$$TT_r = TT_r(\mathbf{X}, \mathbf{Q}) = \sum_i M^T_{ri}(tt_i((\mathbf{MX})_i, Q_i))$$

where tt_i is given by the link performance function (4). Thus

$$\mathbf{TT}(\mathbf{X}, \mathbf{Q}) = \mathbf{M}^T(\mathbf{tt}(\mathbf{MX}, \mathbf{Q})).$$

The set S of supply-feasible (route flow vector, queue delay vector) pairs (**X**, **Q**) is now defined as follows:

$$S = \{(\mathbf{X}, \mathbf{Q}); (\mathbf{MX}, \mathbf{Q}) \in SB\} \tag{8}$$

where SB is given in (6). To state a realistic equilibrium model we need to specify a limitation on **Q**, as the queues and their associated delays cannot be assigned arbitrarily. The most natural limitation to impose is that $Q_i = 0$ if $x_i < s_i$. This queue-equilibrium condition suffices provided there is no blocking back; but if there is blocking back then this queue-equilibrium condition must be made more comprehensive. The most natural condition in this case appears to arise as follows.

We need to restrict **Q** and **X** (or **v** = **MX**) to ensure that queueing can only occur on link i if flow is constrained either by (i) the link i exit saturation flow s_i, or by (ii) an overflow queue arising from a downstream link (or by both (i) and (ii)). This added condition is a "no holding back condition" saying that unless flows are constrained downstream there can be no queue. (For simplicity it may help to imagine here that just the two links j and k are downstream from link i (so we have a simple diverge).) Following these thoughts, the most natural queueing equilibrium condition on (**v**, **Q**) seems to be as follows. For each link i:

(a) $v_i = s_i$, or

(b) $Q_j = MAXQ_j$ for some link j downstream of link i, or (9)

(c) [($Q_j < MAXQ_j$ for all links downstream of link i) and ($v_i < s_i$) and ($b_i = 0$)].

The combined Wardrop / queueing equilibrium condition is now, with condition (9) which allows blocking back:

(**X, Q**) belongs to $[D \times R_+^m] \cap S$;

for each OD pair, routes with longer travel times are unused; and (10)

(**MX**, **Q**) satisfies "no holding back" condition (9) with **v** = **MX**.

As is shown by Daganzo (1998), now

$$[D \times R_+^m] \cap S \text{ is non-empty}$$

does not ensure that there is a solution of the equilibrium condition (10).

6. Equilibration with green times

To obtain a corresponding equilibrium condition with link green-times g_i simply replace s_i by $s_i g_i$ throughout (10).

7. Traffic signal control and routeing context

Webster (1958) was one of the first to seek to model signal timings and their effect on traffic flow at a single junction. Robertson (1969) gives a model of a whole network (TRANSYT) allowing whole network optimisation of traffic signals (for know OD inputs and known routes). Hunt et al (1982) developed the real time control system SCOOT; essentially from the TRANSYT model. The subject is a very large one; Wood (1993) provides a review of certain urban traffic control systems.

Allsop (1974) pointed out the importance of allowing for route choices when considering the impacts of signal control changes. Gartner (1976) considers area traffic control and network equilibrium. Dickson (1981) showed that optimising signals for fixed flows does not give optimum timings when route choices are variable. Signal-controlled networks, allowing route choices to vary, are considered by Fisk (1980) and Sheffi and Powell (1983).

Smith (1987), Van Vuren and Van Vliet (1992), Smith and van Vuren (1993), Yang and Yagar (1995) and Yang (1996) have considered in detail the interaction between signal control and routeing. Meneguzzer (1996, 1997) reports computational experiments with combined traffic assignment and control models and provides a review of models linking signal control and route choice. Hu and Mahmassami (1997) have studied (within a model) day to day evolution of network flows under real-time information and reactive signal control. Taale and van Zuylen (2001) provide interesting discussions of their own work and the work of others combining signal control and route choice.

More recently, Heydecker (2004) and Heydecker et al (2007) propose an adaptive dynamic control system for traffic signals and also considered possible future objectives for traffic signal control. Mounce (2009) has shown that a time-varying equilibrium exists with responsive control (under certain conditions, which prohibit blocking back). Smith and Mounce (2011) present a splitting rate model embracing in a simplified way both traffic re-routeing and signal control adjustments.

LINSIG (2010) software generates signal timings for given flows; this software is often used in real life for junctions and small networks, and may involve small scale routeing considerations.

8. Conclusion

This paper has considered the modelling of traffic control and routeing; seeking to encourage the development, or the further development, of responsive control systems which

(i) make some systematic allowance for travellers' travel choices and

(ii) encourage congestion-reducing travel choices in the future.

A link model which allows for blocking back has also been specified. This may be appropriate when modelling signal controlled junctions, especially when there are short lanes. It would be natural to utilize this link model in the design of responsive control systems, especially when route choice is likely to be involved.

A brief context of academic papers concerning traffic control or route choice has also been provided.

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